Indian J. Phys. **44**, 463–465 (1970)

A note on heat transfer due to the motion of a gradually accelerated plate

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(*Received* 12 *May* 1970)

Heat transfer due to the motion of a gradually accelerated flat plate in a viscous incompressible fluid is considered. Solutions have been obtained in closed form. It is seen that the thermal boundary layer thickness decreases and that the Nusselt number increases with an increase of the Prandtl number.

INTRODUCTION

Leslie (1963) considered the motion of a flat plate from rest in an incompressible visco-elastic liquid proposed by Oldroyd (1950) and obtained the velocity profiles for large and small time using series expansions for the velocity and stresses. Rath (1967) solved the same problem for the fluid in which the coefficient of viscosity depends on the three invariants of the rates of deformation.

We have now considered the problem of heat transfer in a viscous incompressible fluid, with constant fluid properties, due to the motion of a flat plate started from rest with an acceleration $At^{\frac{1}{2}}$. The wall temperature is assumed to be linearly varying with time. Exact solutions have been obtained without considering the dissipation function in the heat balance equation and computations have been carried out for rather small values of Prandtl number in which case we have some justification of neglecting the dissipated heat. Temperature profiles have been drawn for $P_r = 1$, 0.1, 0.01 and some important conclusions have been drawn from the graphs.

HEAT BALANCE EQUATION AND ITS SOLUTION

Here we assume that an infinite flat plate occupies the position $y = 0$ and gradually moves parallel to itself in the x-direction with an acceleration $At^{\frac{1}{2}}$ starting from rest. The energy equation in the absence of dissipated heat becomes (Schlichting 1968)

$$\rho c \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial y^2} \qquad \dots \quad (1)$$

The boundary conditions are

$$\left.\begin{aligned} T(0,t) &= T_w(t) = T_\infty + \lambda t \\ T(\infty,t) &= T_\infty = \text{Constant} \end{aligned}\right\} \tag{2}$$

In the above equations ρ, c and k denote the density, specific heat and conductivity, all of which are assumed to be constants. Also T_w and T_∞ represent the temperatures at the wall and out-side the thermal boundary layer.

We further assume for obtaining similar solutions of the equations (1) and (2), that

$$T - T_\infty = \lambda t\theta(I),\ I = \frac{y}{2(\nu t)^{\frac{1}{2}}} \tag{3}$$

Transformations (3) reduce equations (1) and (2) to

$$\frac{d^2}{dI^2}\theta(I) + 2\,P_r\,I\,\frac{d}{dy}\theta(I) - 4\,P_r\,\theta(I) = 0 \tag{4}$$

where $\quad P_r = \frac{\mu c}{k} = \text{Prandtl number},$

and $\quad I = 0,\quad \theta(I) = 1;\quad I = \infty,\quad \theta(I) = 0 \quad \ldots(5)$

The solutions of (4) and (5) are obtained in a closed form as

$$\theta(I) = 2(\pi)^{-\frac{1}{2}}\{(2P_rI^2+1)\text{erfc}[(P_r)^{\frac{1}{2}}I] - (P_r)^{\frac{1}{2}}I\exp(-P_rI^2)\}$$

where $\quad \text{erfc}[(P_r)^{\frac{1}{2}}I] = \int\limits_{(P_r)^{\frac{1}{2}}I}^{\infty} \exp(-x^2)dx.$

Nusselt Number

We have according to Newton's law of cooling if α is the coefficient of heat transfer and q the quantity of heat exchanged per unit area and unit time,

$$q = \alpha(T_w - T_\infty) = \alpha\lambda t = -k\left[\frac{\partial T}{\partial y}\right]_{y=0} = -k(\lambda t\theta'\tfrac{1}{2}(\nu t)^{-\frac{1}{2}})_{I=0}\left[\theta' = \frac{d\theta(I)}{dI}\right]$$

then $\quad \alpha = -k\frac{\theta'(0)}{2}(\nu t)^{-\frac{1}{2}}$

Thus we obtain a nondimensional heat transfer co-efficient

$$Nu = \frac{2}{k}(\nu t)^{\frac{1}{2}}\alpha = -\theta'(0) = 4\left(\frac{P_r}{\pi}\right)^{\frac{1}{2}}$$

Discussion

In figure 1 we see that there is an increase in thermal boundary layer thickness as the Prandtl number decreases. The Nusselt number, however, increases with

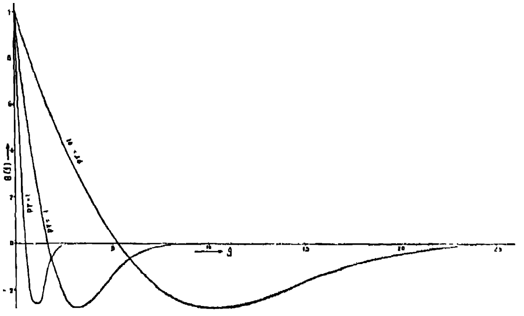

Figure 1. Temperature function.

Prandtl number. It is further seen from the graph that the fluid layer at a distance equal to nearly one-fifth the thermal boundary layer thickness attains the temperature of the fluid outside the thermal boundary layer. In between this layer and the fluid at infinity the fluid temperature falls below T_∞, the maximum fall occurring a little before the point halfway between the wall and infinity.

Reference

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